The basic evolutionary steps in stellar evolution:

- Star formation from a collapsing interstellar cloud
- Energy generation via nuclear reactions
- The cessation of the nuclear reaction and the decay towards the final state
- The possible final states are:
- A White Dwarf, a Neutron Star and a Black Hole
- For $M<M_{ch}$ → a White Dwarf
- For $M>M_{BH}$ → a Black Holes
- For $M_{BH}>M>M_{ch}$ → a Neutron Star

The birth of stars

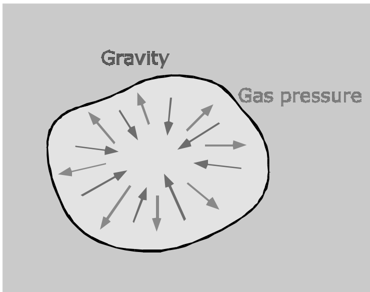
All observational data shows that stars are formed in interstellar clouds.

Interstellar clouds are giant clouds in the galaxy. The clouds contains thousands of solar masses of gas (H, He and metals)

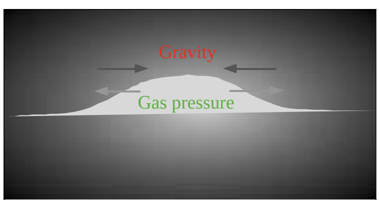
Very young stars are always close, or even inside, interstellar clouds.

The basic idea: an instability in the cloud leads to a collapse of the cloud and eventual formation of a star.

The idea behind the Jean's mass: Minimum mass to collapse



The equilibrium of as cloud



A fluctuation in the density

The Jeans Mass

Mass conservation $$\frac{\partial \rho}{\partial t} + \quad (\rho\vec{v}) = 0$$

Momentum conservation $$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \quad)\vec{v} + \frac{1}{\rho}\vec{\ } p + \vec{\ }\Phi = 0$$

Poisson equation for the gravitational field $$^{2}\Phi = 4\pi G\rho$$

Assume a perturbation

$$\rho = \rho_0 + \rho_1$$

$$p = p_0 + p_1$$

$$\vec{v} = \vec{v}_0 + \vec{v}_1$$

$$\Phi = \Phi_0 + \Phi_1$$

Index: 0 unperturbed

1 perturbed

Assume $X_1 << X_0$

The perturbation equation:

The unperturbed medium is infinite & uniform

$$\frac{\partial^2\rho_1}{\partial t^2} - v_{sound}^2 \nabla^2\rho_1 = 4\pi G\rho_0\rho_1$$

This is the sound propagation eq. But with a source

Assume a solution of the form:

$$\rho_1 = A\exp(\vec{k}\cdot\vec{r} - i\omega t)$$

where: $\omega^2 = v_{sound}^2 k^2 - 4\pi G$

This is the dispersion relation of sound waves in a gravitational field

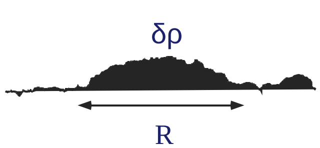
The critical wavelength

$$k_{Jeans} = \left(\frac{4\pi G\rho_0}{v_{sound}^2}\right)^{1/2}$$

$$M_{Jeans} = \frac{4\pi}{3}k_{Jeans}^3\rho_0$$

$$\tau_{dynamic} = (\mathrm{Im}\,\omega)^{-1} \approx \left(4\pi G\rho_0\right)^{-1/2}$$

An estimate of the collapsing cloud



We perturb an infinite gas
The perturbation can be
adiabatic or isothermal (or a mixture of these two extremes)

$$\delta P = c_s^2\delta\rho \qquad c_s = \begin{cases} c_T & \textbf{Isothermal speed} \\ c_a & \textbf{Adiabatic speed} \end{cases}$$

The total mass of the perturbation is:

$$M = \frac{4\pi}{3} R^3 \delta\rho$$

The gas pressure depends only on the gas properties.

The pressure drop (the acting force, the pressure difference) depends on the dimension R. Gravity increases with R For sufficiently large R Gravity wins.

There exists Rcrit so that for all R>Rcrit gravity wins

All clouds with masses greater than Mjeans collapse

A simple estimate of the Jeans mass

the pressure difference δP acts per unit area

The difference of the gravitational force is: $\frac{G(4\pi/3)R^3\delta\rho}{R^2}$

The mass per unit area is: $(\rho+\delta\rho)R \approx \rho R$

The critical dimension is obtained by the equation:

$$\delta P = c_s^2 \delta\rho \quad \frac{G(\frac{4\pi}{3}R^3\delta\rho)(\rho R)}{R^2}$$

$$R_{crit} = R_{Jeans} = \frac{c_s}{\sqrt{\frac{4\pi}{3} G\rho}}$$

The typical time scale is:

$$\tau = \frac{1}{\sqrt{\frac{4\pi}{3} G\rho}}$$

This is the typical time scales for changes in the self-gravitating object

Typical times:

The sun $\bar{\rho} = 1.4 gm/cc \quad \tau = 1.6 \times 10^3 \sec$

Neutron star $\bar{\rho} = 10^{15} gm/cc \quad \tau = 6 \times 10^{-5} \sec$

Interstellar cloud $n = 1 p/cc \approx 1.7 \times 10^{-24} gm/cc$

$$\tau_{Cloud} \approx 1.5 \times 10^{15} \sec = 4.7 \times 10^7 yr$$

In a star $R_{Star} = R_{Jeans}$

For an interstellar cloud at T=10K

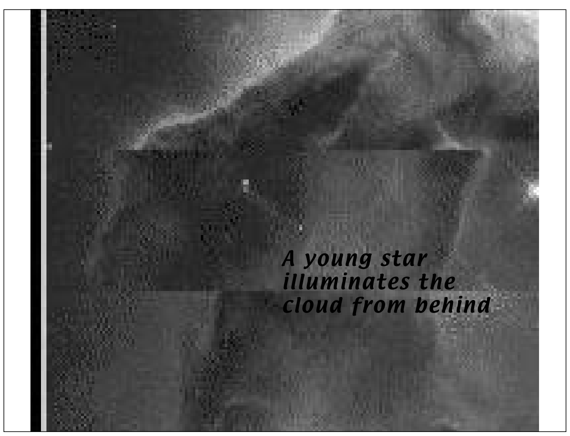
$$R_{Jeans} = c_s \tau = 3\times10^4 cm/\sec\ 5\times10^7 yr$$
$$= 4.3\times10^{19} cm = 45 lyr$$

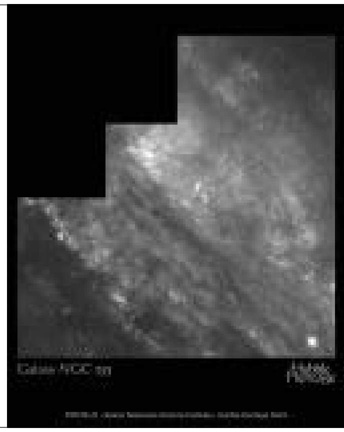
$$M_{Jeans} = \frac{4\pi}{3} R_{Jeans}^3 \rho$$
$$= 3.2 \times 10^{35} gm = 160 M_{sun}$$

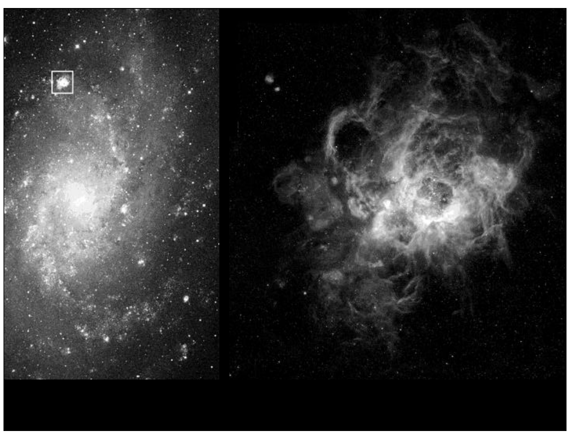
The Jeans mass in the interstellar medium is too high to form a star like the Sun

The large cloud must fragment during the collapse to smaller clouds

The fragmentation process may cause double stars, triplets etc.







Typical conditions in the interstellar clouds are:

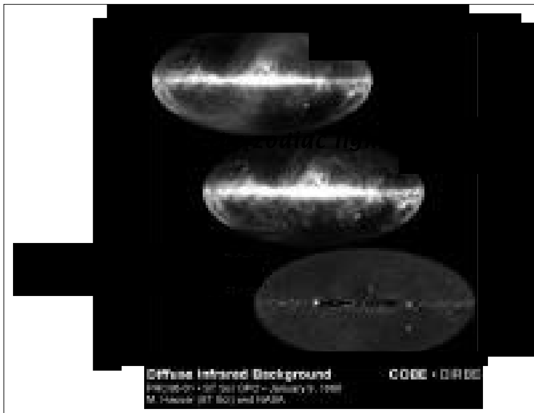
Density ~ few to hundreds particles/cc

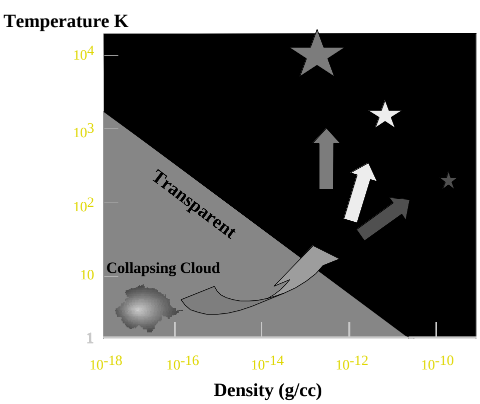
Temperature ~ 10-100K

Size ~ few thousands of solar masses

Hence the radiation from the interstellar cloud should be in the Far Infra-red

The clouds are strong emitters in IR





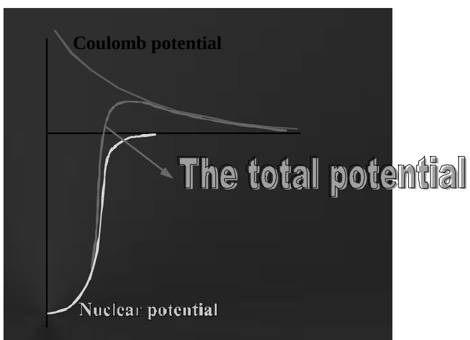
The collapsing cloud is first transparent and then becomes opaque. At this moment it is a new star.

The new star is very large and relatively cool, hence it shines, looses energy and contracts.

The new star is an ideal gas, hence it heats upon contraction.

The contraction continues till nuclear reactions are ignited and supply the energy lost by the star from its surface

In the nuclear reactions
H is fused into He
He fuses into C^{12}
C^{12} fuses into Mg^{24}
How far can it go?



The nuclear potential has a finite range while the Coulomb potential has an infinite range

The stages of the nuclear reactions in stars

Hydrogen burning

(a) Hydrogen is the most abundant specie in the cosmos

~ 0.7 by mass

(b) The smallest Coulomb barrier

Because of (b) at each step only the isotope with the smallest Z (atomic number) will react

The temperature in the star determines which reaction 'goes'. Always the 'easiest' reaction is the first to go/

The natural way to continue is $He^4 + He^4 \Rightarrow Be^8$

But Beryllium 8 does not exist in Nature!

This problem was a major problem in nuclear Astrophysics in the early 50's and today in Big Bang nucleosynthesis

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Helium reactions **Energy**

$2He^4 \longrightarrow Be^8 + \gamma - 95kev$

Be^8 is unstable and decays after 10^{-17} sec.

Be^8

96keV

$He^4 + He^4$

When T ~ 10^8K a particles from the tail of the MB distribution have sufficient energy to reach the unstable ground state of Be^8 (Salpeter)

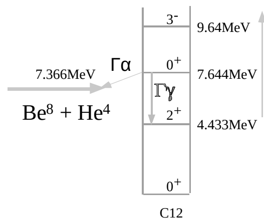
$He^4 + He^4 \Leftrightarrow Be^8 + \gamma + Q$ **Equilibrium**

A small amount of Be^8!

Once Be^8 exists (even for a short time) we get:

Be^8 + He^4 ⟹ C^{12*} - 286keV

The free particle reaches the Carbon in an excited state. The decay back of the state is back to the incoming channel and little to a lower state and down to the ground state of Carbon.



This is the Nobel prize in astrophysics: Salpeter

The Carbon-Carbon Reaction

$$C^{12} + C^{12} \quad \left(Mg^{24}\right)^*$$

$$\left(Mg^{24}\right)^* \quad Na^{23} + p \ +2.24MeV$$

$$Ne^{20} + He^4 + 4.616MeV$$

$$Mg^{24} + \gamma + 13.93MeV$$

$$Mg^{23} + n - 2.605MeV$$

$$O^{16} + 2He^4 - 0.114MeV$$

Because of the need to penetrate a greater and greater barrier every time a given nuclear fuel is depleted, the star resumes it contraction (because it is hot and losses energy from the surface).

As the star contracts, its temperature rises.

The radiation field is in equilibrium with the matter.

As the temperature rises, the mean energy of the photons increases.

When **hν = kT~Enuclear-bind**

the γ has sufficient energy to break the nuclei.

For sufficiently high temperatures molecules disintegrate

At still higher T, the individual atoms lose their electrons

At T~ Enuclear-bind/k the nuclei disintegrate

The α capture process

$C^{12} + He^4 \rightarrow O^{16} + \gamma$ (7.162MeV)

$O^{16} + He^4 \rightarrow Ne^{20} + \gamma$ (4.73MeV)

$Ne^{20} + He^4 \rightarrow Mg^{24} + \gamma$ (9.317MeV)

….. $\rightarrow$

At high temperature the photons start to disintegrate the matter and particles are chipped off the nuclei.

These particles are absorbed by the remaining nuclei.

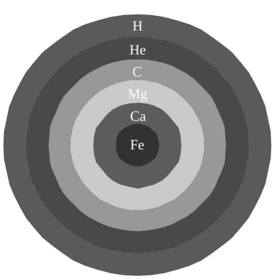
The α capture continues till Fe^{54}

Summary of nuclear history:

- **At T~ 10^7K: H $\rightarrow$ He. Energy released: 6.1×10^{18}erg/gm**
- **At T~10^8K: He $\rightarrow$ C^{12}: the 3α reaction. Energy released 9×10^{17}erg/gm**
- **At T~6×10^8K: Carbon $\rightarrow$ Magnesium.**
- **At T~ 10^9K: The radiation starts to disintegrate the nuclei a capture process starts.**
- **At T~4×10^9K: most of the matter becomes Fe^{54}.**
- **At T~ 5×10^9K: The radiation field disintegrates the Fe into α particles and neutrons.**

There is no more nuclear fuel: Radiation destroys matter

The structure of the star before photon disintegration starts



The binding energy of Fe^{54} is about 8Mev per nucleon.

The photons must invest energy to break the Fe nucleus

The energy is taken from the gas

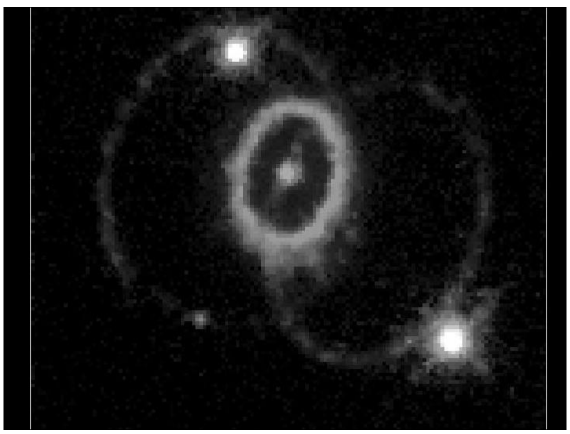
But:

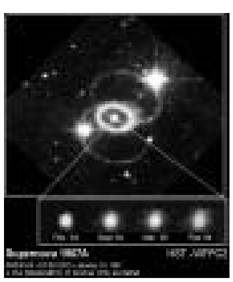
$$P = \frac{energy}{volume}$$

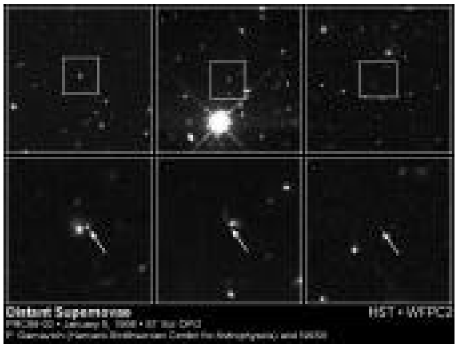
So the energy reduces the pressure of the gas.

The star losses it support against gravitation and collapses. This collapse lead to supernova - an explosion of the star which releases about 10^{53} ergs.

A galaxy emits 10^{45}ergs/sec, so a SN can supply the entire galaxy luminosity for 10^{8} sec, or about 3 years







How the structure of the star react to the nuclear reactions?

In the conversion of H into He the star losses 4 protons and gains one He nucleon.

The pressure of an ideal gas depends on the internal energy which is 3/2kT per particle.

As H converts into He, the number of particle decreases and so does the pressure.

The pressure in the core decreases and the weight of the outer layers presses the core to higher densities and temperatures.

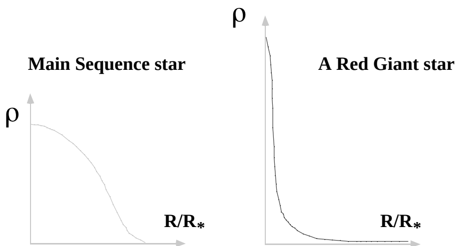
As the core contracts, the outer layers expand.

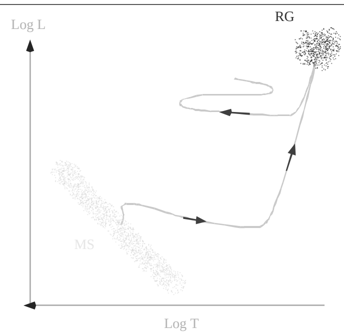
The star becomes a Red Giant: very dense core and very extended envelope.

Typical density distributions in a MS star and a RG

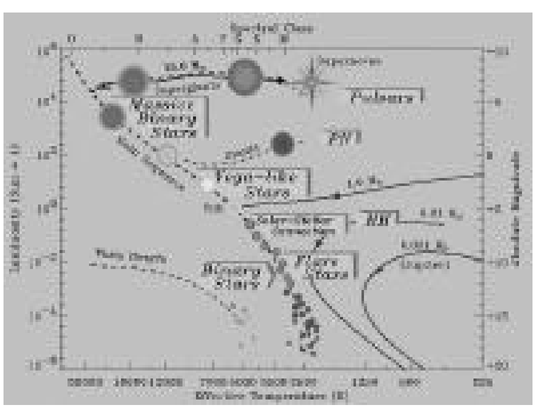
$R_{Red\ Giant} = 1000\ R_{Sun}$

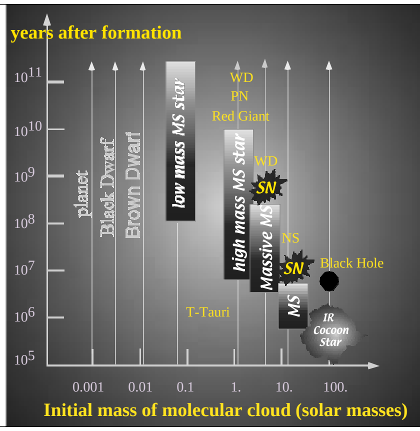
In the figure the radii are normalized





A typical path of a star in the HR diagram





The saga of stars

As the star contracts, it heats and losses energy from the surface

Gravitational energy gives very short lifetime.

Nuclear energy releases energy via fusion - building more and more massive nuclei

Whenever a fuel is depleted gravity wins and contraction resumes

This process continues until we reach the bottom of the binding energy per nucleon (Fe). This occurs when T is so high that the γ's disintegrate the nuclei. As a consequence the gas losses its pressure and the star collapse.

The nuclei disintegrate back into H, n and He!

All the stellar nuclear evolution is reversed.

The nuclear reactions halt the eventual collapse

but do not prevent it.

Gravity always wins

(in massive stars)